Pentti Kaitera

On the influence of drainage basin on discharge maximum in springtime

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COMMISSION DE POTAMOLOGIE
Question 2 — Rapport 1

ON THE INFLUENCE OF DRAINAGE BASIN ON DISCHARGE MAXIMUM IN SPRINGTIME.

BY PENTTI KAITERA
(Finland).

The present article is a short summary of a more extensive report on a research the object of which has been snow melting in springtime and its influence on run-off in the basins of water courses in Finland. The significance of only the following factors is examined here : the form of the basin, the area of the basin, the percentage of lakes, and that of arable ground.

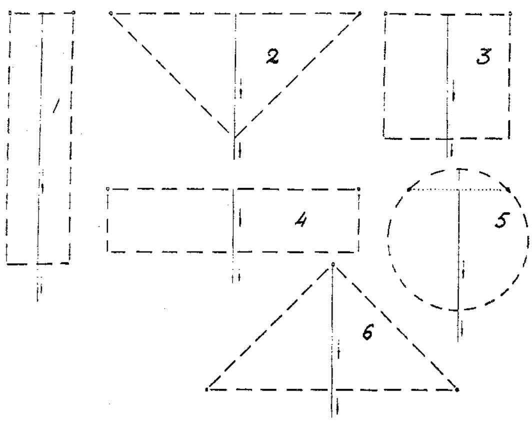


Fig. 1. — Different forms of basin (cases 1-6).

In the drainage basins 1-6 in figure 1, which basins are of equal size, but dissimilar in form, the run-off is supposed to be constant (v units of length in one of time) and water to flow perpendicularly against the main water courses. Further, the summed-up supply (1)

(1) The *summed-up supply* is equal to water released by melting of snow and of the frozen ground plus precipitation.

(u) is supposed to be constant. Thus we get the equation :

$$q = vu \int_0^t f(vx)\,dx, \tag{1}$$

where q = the discharged quantity of water after the time t. If the summed-up supply is a function of time $u = \varphi(t)$, the equation of q is a double integral of $\varphi(t - x) . f(vx)$.

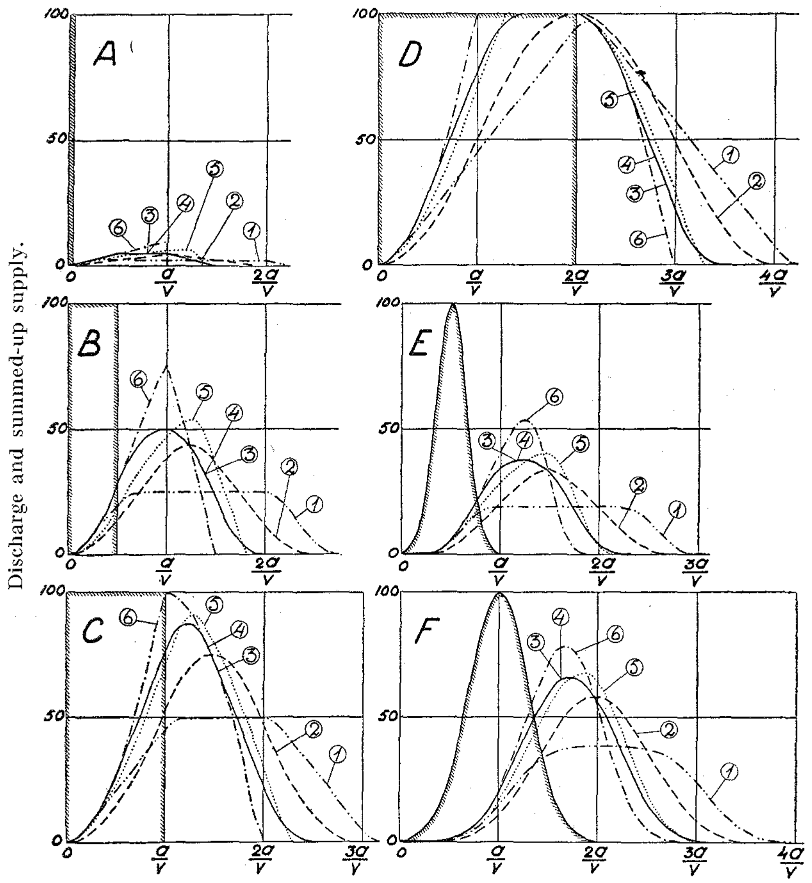


Fig. 2. — Run-off waves in basins 1-6 (*fig.* 1) caused by different summed-up supply (cases A-F). Ordinates : Discharge and summed-up supply; abscissae : time (= length divided by velocity).

If we suppose the summed-up supply to change in the cases A-F according to figure 2, analytically we get the corresponding run-off waves in the basins 1-6, which are also set forth in figure 2. The curves in figure 2 illustrate the dependence of the run-off of the summed-up supply and of the form of the basin. The faster the run-off, the greater is generally the discharge maximum. Where the run-off of the summed-up supply continues a sufficiently long time, the discharge maximum will equal the maximum of the summed-up supply and becomes independent of the form of the basin, as well as of its other qualities, *e.g.* the area of the basin.

We write

$$\mu = \frac{F}{l^2}, \tag{2}$$

where :

μ = basin form coefficient,
F = the area of the basin, and
l = the distance of the extreme point of the basin from the place of measurement. (In natural channels this is equal to the length of the main water course.)

Employing the following type of formula for determining the discharge maximum

$$Hq = \mu^{\eta} k Hu + c, \tag{3}$$

where :

Hq = discharge maximum,
Hu = the maximum of the summed-up supply, and η, k, and c constants dependent on the summed-up supply and the nature of the basin, and substituting the values in casu of η, k, and c in the formula, we shall get Hq-values, which are approximately equal to the values arrived at when using the formula (1). *E.g.* in the case (*fig.* 2), writing $\eta = \frac{1}{6}$, $k = 1.0$, and $c = 0$, and Hu being 100, we get, using formulas (1) and (3), the following Hq-values :

Basin	Formula (1)	Formula (3)
1	50	76
2	75	79
3	88	87
4	88	87
5	91	90
6	100	100

The influence of the area of a basin (F) can be illustrated by substituting *e.g.* in the formula (3)

$$k = \frac{k_1}{\sqrt[n]{F}}, \qquad \text{where } n \geqq 2. \tag{4}$$

Among the empirical formulas in use in Finland for computing the discharge maximum, the following may be mentioned

$$Hq = 10 + \frac{8L_1}{1 + 0.02\,J^2 + 0.01\,\sqrt{F} + 0.0001\,J^2\,\sqrt{F}} \tag{5}$$

(Renqvist 1933), where

Hq = discharge maximum ($l/s.km^2$).
L_1 = the average maximum of the daily decrease of the water content of snow (mm),
J = percentage of lakes, and
F = the area of the basin (km^2); or

$$MHq = 10\left(1 - \frac{J+I}{100}\right)^8 \frac{1}{\sqrt[6]{F}} + 0.03. \tag{6}$$

(Hallakorpi 1934), where

MHq = average discharge maximum ($l/s.ha$),
J = percentage of lakes,
I = index of infiltration, and
F = the area of the basin (ha).

It has been generally accepted that the run-off of rainfall takes place faster on arable land than on forested ground. Supposing

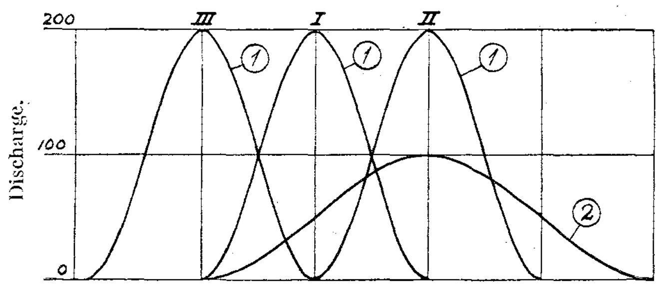
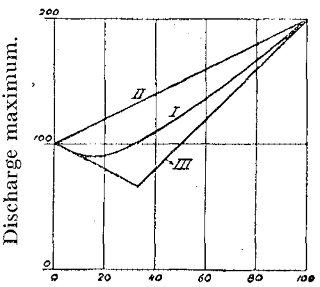


Fig. 3. — Sine curves assumed to illustrate run-off.

the run-off on arable ground to require one half of the corresponding time on forested ground, and that a sine curve represents a run-off wave (*fig.* 3), and placing, with reference to time, the curve representing arable land (Curve 1) in positions I, II and III in comparison with the curve for forested ground (Curve 2), we find that the discharge maximum changes as illustrated in figure 4, when forested ground gradually turns into arable one. Generally, excepting the case III, the discharge maximum diminishes in the beginning, but gradually starts growing. This is true both in cases, in which the discharge maximum of arable ground is equal to that of forested land, and in those where it is greater, snow melting earlier in open fields than in forests, which has the same effect as faster run-off. In the former case, the minimum occurs when the percentage of arable ground is 50.



It is assumed: 1, that the maximum run-off from cultivated land is twice that from forested land and 2, that the spring flood caused by the melting passes by in half the time, if the area is cultivated. The curves I, II and III correspond to different beginning moments. Total run-off constant.

Percentage of arable ground.

Fig. 4. — Change in discharge maximum when forested land is changing into cultivated land. Cases I, II and III correspond with cases in figure 3.

On run-off maximum in springtime. — The run-off maximums in springtime are studied in 47 drainage basins, the location of which is given in figure 5. Table 1 contains the numbers of the basins, and

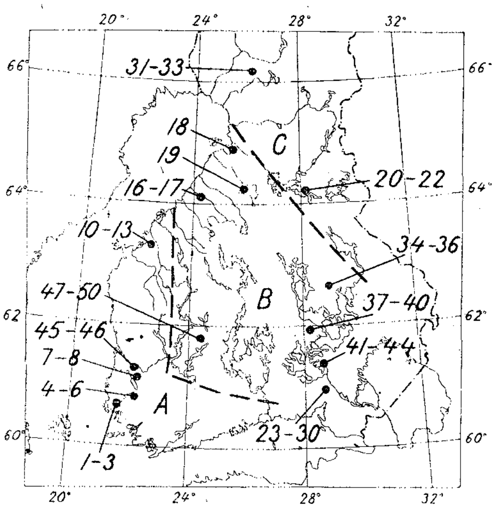


Fig. 5. — Location of drainage basins.
Figures refer to the numbers of basin in table 1.

their area (F), percentage of lakes (J), mean discharge maximum in 1934-1937 (MHq), and the average maximum decrease of the water content of snow in five days (L_5) during these years. Further, the basin form coefficient (μ), and the percentage of arable ground (Z) are given in the basins poor of lakes ($J = 0\text{-}3\ ^0/_0$), where the percentage of arable ground is known.

TABLE 1.

Drainage basins examined, their area (F), percentage of lakes (J), basin form coefficient (μ), percentage of arable ground (Z), average discharge maximum in springtime (MHq), maximum decrease of the water content of snow in five days (L_5), and the difference of the date of flood crest in spring and the date on which snow in open fields has practically disappeared ($T_1 - T_0$).

No of basin	F (km^2)	J (%)	μ	Z	MHq 1934-37 ($l/s.km^2$)	L_5 (mm)	$T_1 - T_0$ (days)
1.......	94	1	0.3	40	100	26	1
2.......	79	2	0.4	39	107	26	0
3.......	29	3	0.3	40	91	26	—2
4.......	90	1	0.3	2	93	29	2
5.......	48	1	0.2	1	110	28	2
6.......	37	1	0.1	2	106	29	—3
7.......	195	8	—	—	44	26	—4
8.......	150	10	—	—	24	25	7
10.......	109	0	0.3	33	114	46	8
11.......	78	1	0.3	31	113	36	7
12.......	59	1	0.3	31	117	36	7
13.......	28	1	0.4	22	110	37	7
16.......	67	2	0.1	10	127	49	2
17.......	60	2	0.1	7	127	49	2
18.......	13	0	—	—	150	45	0
19.......	10.5	0	0.2	21	160	53	0
20.......	48	6	—	—	132	69	4
21.......	26	9	—	—	138	69	6
22.......	4	8	—	—	145	70	2
23.......	177	2	0.2	27	93	41	—2
24.......	12.5	8	—	—	49	32	0
25.......	116	1	0.2	27	107	35	—2
26.......	45	0	0.2	32	102	32	—3
27.......	7.5	0	0.6	39	161	35	—4
28.......	13.5	3	0.2	18	128	36	—2
29.......	14.5	4	—	—	103	34	0
30.......	13	1	0.3	29	106	33	—5
31.......	15.5	0	0.2	1	244	87	—1
32.......	76	2	0.3	1	299	89	0
33.......	9	13	—	—	186	87	1

TABLE 1 (*suite*).

No of basin	F (km^2)	J (%)	μ	Z	MHq 1934-37 ($l/s.km^2$)	L_5 (mm)	$T_1 - T_0$ (days)
34	129	9	—	—	46	58	6
35	20	1	0.2	3	109	58	—3
36	23	9	—	—	87	58	2
37	66	28	—	—	11	48	9
38	37	21	—	—	17	49	9
39	25	19	—	—	20	50	8
40	6.5	22	—	—	19	49	8
41	4.5	0	0.3	10	115	60	—4
42	4.5	0	0.4	20	110	60	—3
43	10	1	0.3	2	180	60	—2
44	4.5	0	0.4	0	178	60	—2
45	7	0	0.3	75	250	48	—4
46	8	1	0.3	41	213	46	—5
47	60	11	—	—	57	63	8
48	29	11	—	—	90	63	4
49	16	10	—	—	75	63	1
50	9	18	—	—	50	63	11

The basins have been divided into three groups according to their size, the limits being 4-29 km^2, 30-79 km^2, and 80-195 km^2, and according to the percentage of lakes also into three groups with 0-3 %, 4-13 %, and 18-28 % as limits. Table 2 gives the group means.

TABLE 2.

Discharge spring maximum (Hq), maximum decrease of the water content of snow in five days (L_5), and the transformed discharge maximums in various basins based on them. F = area of drainage basin, J = percentage of lakes.

Group	J (%)	F (km^2)	Number of basins	Number of annual values	Hq ($l/s.km^2$)	L_5 (mm)	Transformed Hq($l/s.km^2$) for L_5=54mm
1	0.8	13.5	15	52	147	49	155
2	1.3	61.0	9	36	134	42	154
3	0.9	110.3	5	16	101	34	128
4	8.8	16.0	8	28	113	59	107
5	7.7	52.0	2	6	107	67	94
6	9.0	158.0	3	12	38	36	47
7	20.0	14.4	3	10	26	52	27
8	24.5	51.5	2	8	14	49	15

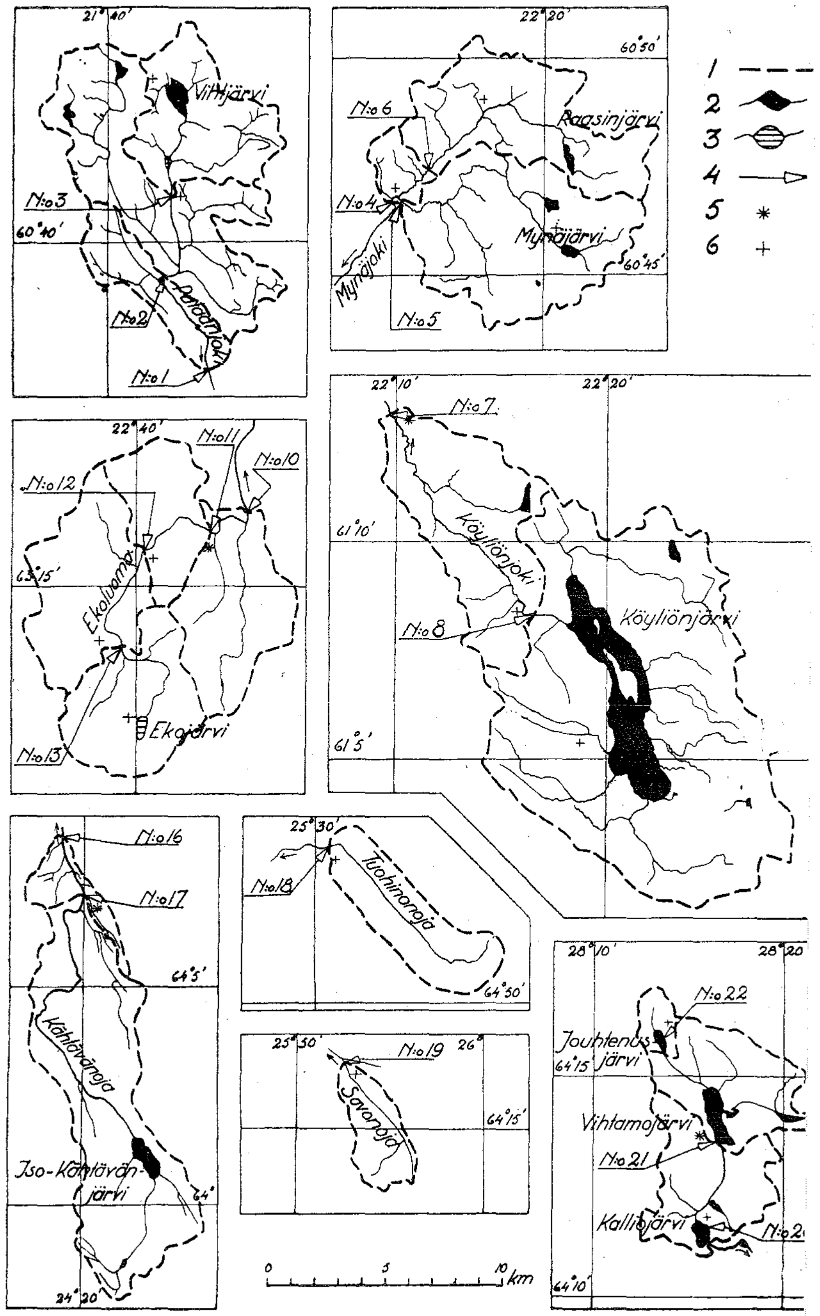
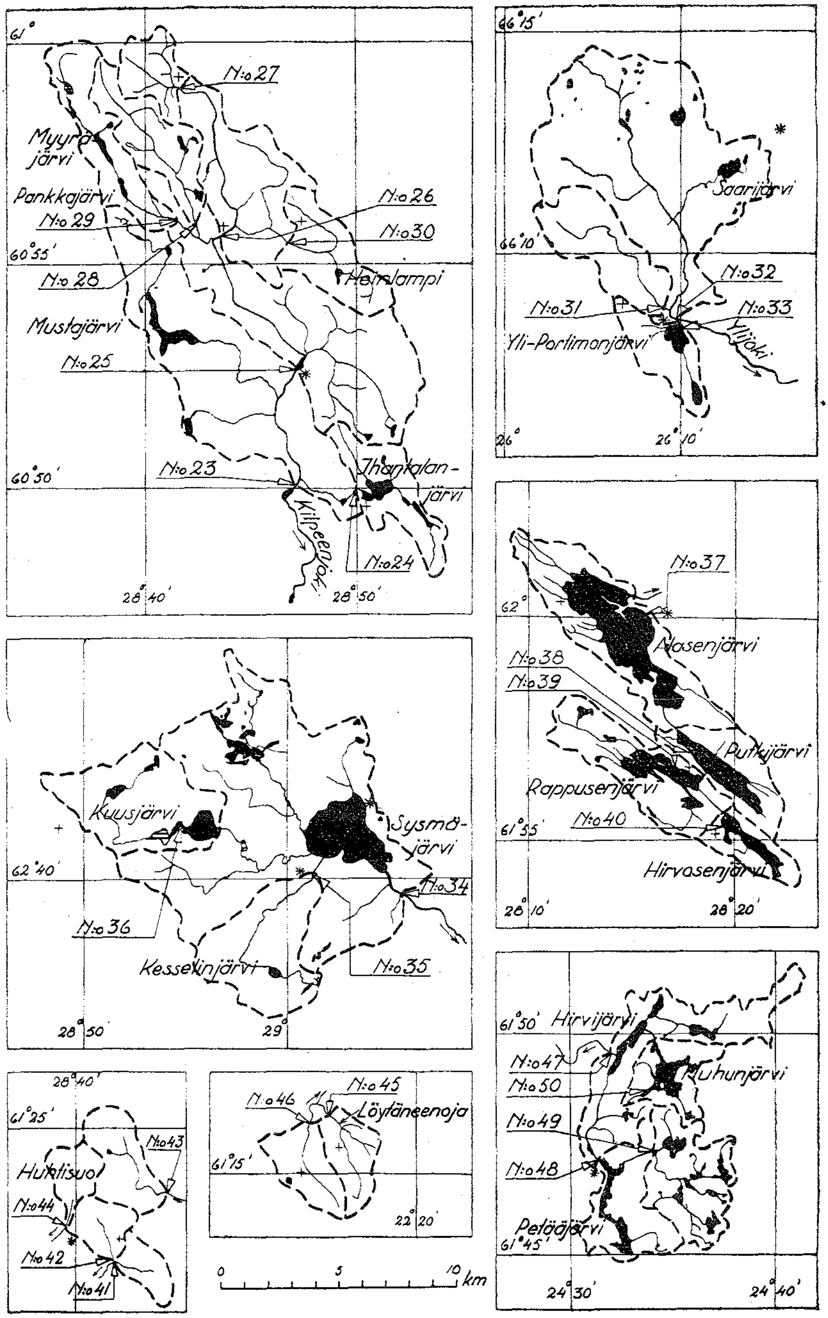


Fig. 6. — Charts of draina

Explanation : 1, Boundary of drainage basin; 2, Lake; 3, Tempora



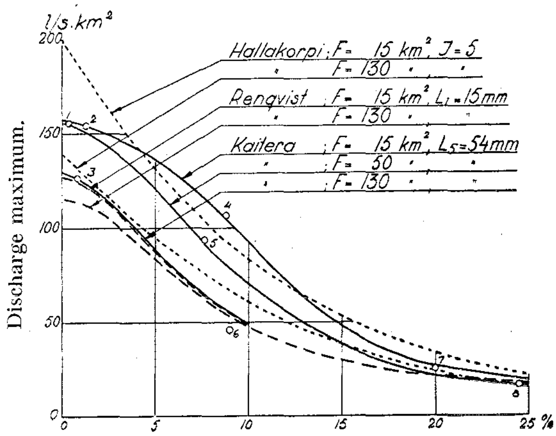
basins listed in table 1.

lake; 4, Gauge; 5, Snow station; 6, Precipitation station.

The discharge maximums corresponding to $L_5 = 54$ *mm* have been obtained by transforming supposing linear dependencies

$$\frac{Hq\,(\text{for } L_5 = a)}{Hq\,(\text{for } L_5 = b)} = \frac{a + 40}{b + 40}.$$

In figure 7 we have the smoothed out curves based on the transformed group values. The curves elucidate the dependence of the Hq-values on the percentage of lakes when $F = 15$, 50, and 130 km^2. The curves determined by means of the formulas (5) and (6), and approximately corresponding to the L_5-value 54 *mm*, $y = 5$ and when $F = 15$ and 130 km^2, are also given in figure 7.



Percentage of lakes.

Fig. 7. — Influence of lake percentage on discharge maximum.

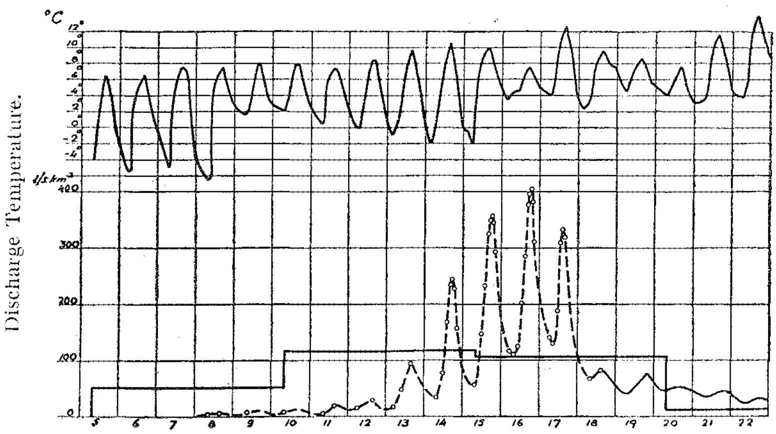
The formulas (5) and (6) are based on data from drainage basins with sizes varying from 240 to 62100 km^2, and a percentage of lakes between 1-23 %. If we, using these types of formulas as a starting point, work out formulas giving values that closely follow those in Table 1, we get *e.g.* the following formulas

$$Hq = \frac{1.5\,(L_5 + 40)}{0.5 + 0.01\,J^2 + 0.05\,\sqrt{F} + 0.0005\,JF} + c, \tag{7}$$

and

$$Hq = \frac{3.5\left(1 - \dfrac{J}{L_5 + 40}\right)^8 (L_5 + 40)}{\sqrt[6]{F + 1}} + c, \tag{8}$$

in which Hq = spring maximum of the daily average run-off ($l/s.km^2$), F = area of the drainage basin (km^2), J = percentage of lakes, and L_5 = maximum decrease of the water content of snow in five days (*mm*). When $c = 0$, the formulas give too great values for basins in group 1 (Table 2). This is probably due to the fact that the run-off maximum in these basins approaches the maximum summed-up supply, resulting in the exemption of the influence of the area (*cf. fig.* 2). This explanation is supported also by the fact that the transformed values of Hq in groups 1 and 2 in Table 2 are practically equal in spite of the considerable difference in the size of the basins. Besides, in many basins in group 1 daily fluctuations of the run-off occur during the melting season, due to the great variations of the daily temperature of the air (*fig.* 8). These fluctuations can be so great that the daily maximum may be 3-4 times the minimum. Similar fluctuations of the daily run-off are not noticeable in the other groups. This is an additional proof of the fact that the maximum of the daily average run-off in group 1 depends on the quantities of water released by melting of snow during the day in the basin.



April, 1937.

Fig. 8. — Fluctuations of the run-off during the melting season in basin 45 (Table 1) in 1937.

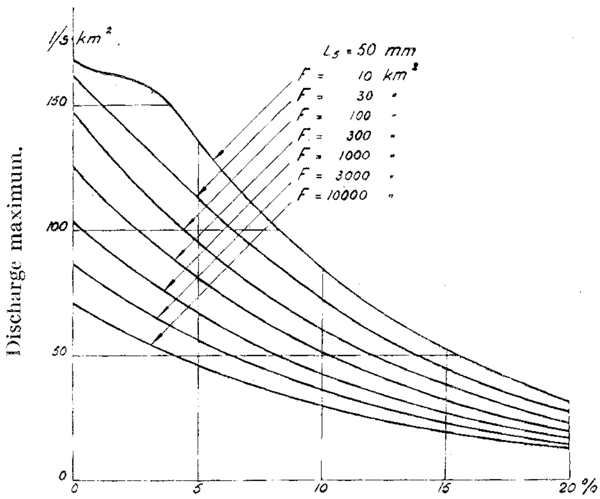
If we write *e.g.*

$$c = 3 - \frac{L_5 + 40}{1.5 + 0.004F^2 + 0.6J^2}, \tag{9}$$

and substitute it in the formulas (8), we get also in group 1, on an average, the values in Table 1. The mean deviation of the values computed by the formulas (8) and (9) is about 30 %.

The formula (8) is also applicable to large basins, c then being = 3.

The mean deviation of the computed values (in 1920-1935) in 21 basins with areas varying between 1035-62930 km^2, and with a percentage of lakes between 1-21 %, has been about 35 %. The same accuracy is obtained by the fromula (5). Figure 9 gives the computed values according to the formulas (8) and (9) with varying F-values, and writing $L_5 = 50$ *mm.*



Percentage of lakes.

Fig. 9. — Discharge maximum according to the formulas (8) and (9) with varying F and assuming $L_5 = 50$ mm.

The influence of arable ground has been examined by dividing the 28 basins in Table 1, the percentage of arable ground of which is known, and that of lakes varying between 0-3 %, into four groups according to the percentage of arable ground, and by computing the group means. The means of the groups, the limits of which are 0-3 %, 7-22 %, 27-33 % and 39-75 % are as follows

	Percentage of arable ground			
	2 %	15 %	30 %	43 %
Number of basins	8	7	7	6
Number of years of observation.	30	26	24	20
F (km^2)	39.7	28.6	75.4	43.4
J (%)	0.9	1.2	0.8	1.3
L_5 (mm)	54	48	36	32
Hq ($l/s.km^2$)	164	126	108	138
Transformed Hq ($l/s.km^2$)	147	120	127	175

The transformed Hq — values representing the discharge maximum of basins with an area of 40 km^2, and a percentage of lakes of 1 %, when $L_5 = 45\ mm$, have also been computed employing the formulas (8) and (9). The relative values, compared with those of the first group, are as follows

Percentage of arable ground	Relative discharge maximum
2	100
15	82
30	86
43	119

Thus we find that the discharge maximum of the basins approximately changes according to the diagrams I in figure 4 with growing percentage of arable ground. The location of the arable ground

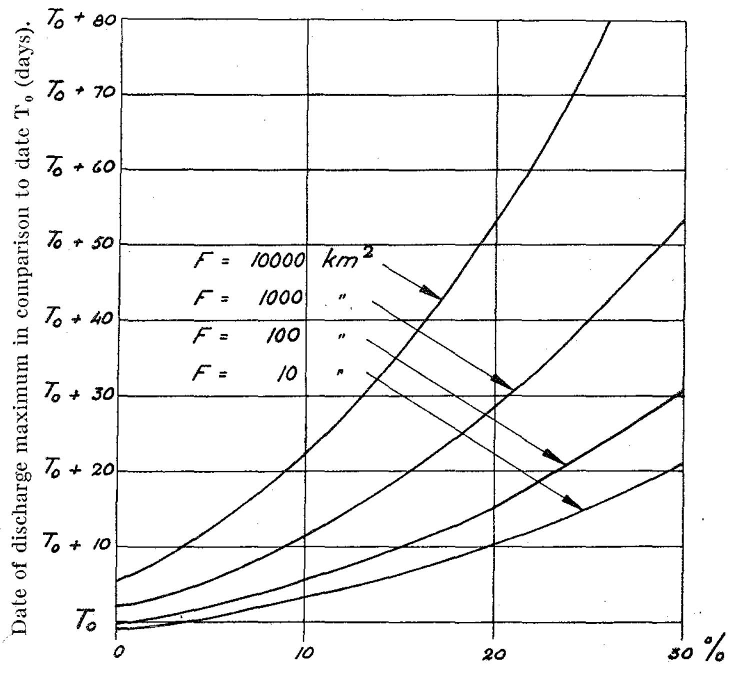


Fig. 10. — Date of discharge maximum, computed by formula (10) in comparison to date T_0.

is also an influential factor. Generally, most arable ground in Finland lies along the water courses and mainly at the lower ends of them, which fact strengthens the general development above.

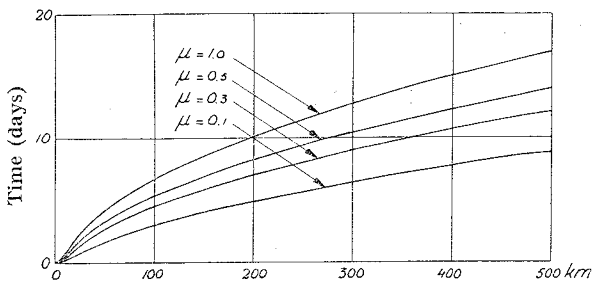
Date of discharge maximum during Melting season. — The difference of the date of the discharge maximum during the melting season (T_1) and the date (T_0), *i.e.* the date on which snow in open fields has practically disappeared, is given in Table 1. Knowing the area of the basin, and its percentage of lakes, this difference of dates can be computed employing *e.g.* the following formula

$$T_1 - T_0 = \sqrt[4]{0.5F + 10}(0.008J^2 + 0.12J + 1) - 3, \tag{10}$$

where $T_1 - T_0$ is expressed in days, and F in km^2. This formula is also applicable to large basins in Finland. The mean deviation of the computed dates is $\pm$ 3 days for basins listed in Table 1, and for large basins $\pm$ 9 days. Figure 10 gives the differences computed employing the formula (10) when F = 10, 100, 1000, and 10 000 km^2, and J = 0-30 %. If we determine the difference (t) of the dates of dis-charge maximums in two lakeless basins according to the formula (10), we get

$$t = \sqrt[4]{0.5F_1 + 10} - \sqrt[4]{0.5F_2 + 10}, \tag{11}$$

where F_1 and F_2 denote the area of the basins in question. If we substitute F_1 from Table 2 in the formula (11), and write $F_2 = 0$, and compute the differences (t) corresponding the μ-values 0.1, 0.3,



Distance from the upper end of the water course.
Fig. 11. — Advance of flood crest in lakeless basins in Finland (μ = form coefficient, formula 2).

0.5 and 1.0, we get the curves in figure 11. They should illustrate the rate of advance of flood crest in springtime in lakeless basins with varying form. The advance of the flood crest is faster at the lower end of a water course, contrary to the case in certain rivers in Central Europe. They descent more gently at their lower course and waters from mountains at their upper course determine the nature of the advance of the flood crest. In Finland there is often hardly any difference in the decline of the lower and the upper parts of a water course, neither in the snow cover along them, this being the case especially in water courses poor of lakes, and snowmelting takes place simultaneously in large regions. Thus the point in Finland is not so much a flood crest advancing down a water course from the fountain end of it in springtime, than the sum of numerous flood crests advancing simultaneously from the various parts of the basin, which being the case, the area of the basin becomes the decisive factor.

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